Department of Communications Engineering

Communication Systems

Third Year Class

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Lecture 8

Angle Modulation Bandwidth

FM signal Bandwidth :

we have shown that FM Bandwidth depends on the modulation index m_f.

$$S_{FM}(t) = A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right]$$
$$+ A J_2(m_f)\left[\cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t\right]$$
$$+ A J_3(m_f)\left[\cos(\omega_c+3\omega_m)t - \cos(\omega_c-3\omega_m)t\right] \ldots$$

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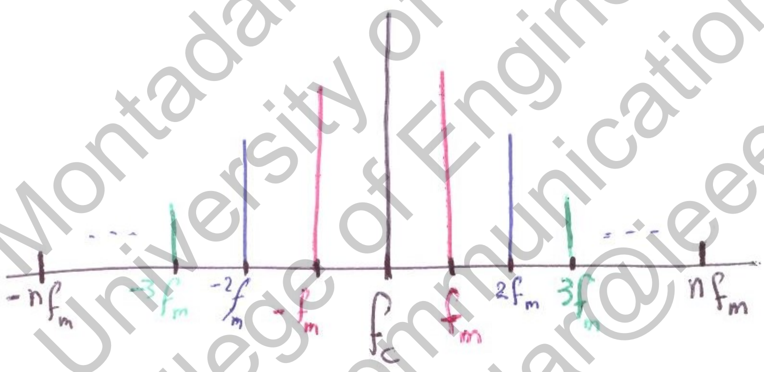


frequency spectrum of FM signal for n significant sidebands.

: Thus, to calculate the bandwidth, we need to consider n sidebands.

* In this case, n must not be very large, we can consider the sideband of amplitude which is 1% of the unmodulated carrier's amplitude (A_c).

In other words sideband amplitude $= 1\% A_c$ to be considered.

Carson's Rule :

As a rule of thumb, for single-tone FM signal, and as an approximation :

$$B_{FM} = 2(\Delta\omega + \omega_m)$$

since $\Delta\omega = m_f \omega_m$

$$B_{FM} = 2(m_f \omega_m + \omega_m)$$

$$\therefore \boxed{B_{FM} = 2\omega_m(m_f + 1)}$$

Two cases can be considered :

(1) $\Delta\omega \ll \omega_m \rightarrow m_f \ll 1$ (Narrowband FM)

$\boxed{B_{FM} = 2\omega_m}$ similar to AM signal

(2) $\Delta\omega \gg \omega_m \longrightarrow m_f \gg 1$ (wideband FM)

$$\boxed{B_{FM} = 2\Delta\omega}$$

EX. Find the bandwidth of a commercial FM transmission if the frequency deviation = 75 KHz and modulated by a 15 kHz signal.

Solu. $BW = 2(\Delta f + f_m) = 2(75+15) = 180\,KHz$

EX. Determine the bandwidth of a narrowband FM signal which is generated by a 4 KHz audio signal modulating a 125 MHz carrier.

Solu. since it is a narrowband FM, $BW = 2f_m = 8\,KHz$.

EX. the maximum deviation allowed in an FM broadcast system is 75 KHz. If the modulating signal is a single-tone sinusoid of 8 KHz, determine the bandwidth of the FM signal. What will be the bandwidth when the modulating signal amplitude is doubled?

Solu. we have given $\Delta f = 75\,KHz$, $f_m = 8\,KHz$.

$BW = 2(\Delta f + f_m) = 2(75+8) = 166\,KHz$

Now, we know $\Delta f = m_f f_m \rightarrow \Delta f = k_f V_m f_m$

if $V_{m_{new}} = 2V_{m_{old}}$

$\therefore \Delta f = 2\underbrace{k_f V_m}_{m_f} f_m = 2 m_f f_m$

$\therefore \Delta f_{new} = 2\Delta f_{old} = 2 * 75 = 150\,KHz$

$\therefore BW_{Fm} = 2(150+8) = 316\,KHz$.

PM Modulation (single-tone PM)

The instantinuous phase of PM is

$$\phi_i = \omega_c t + k_p x(t)$$

and for a single-tone modulating signal,

$$x(t) = V_m \cos\omega_m t$$

thus,

$$\phi_i = \omega_c t + \underbrace{k_p V_m}_{\theta_d} \cos\omega_m t$$

$$\phi_i = \omega_c t + \theta_d \cos\omega_m t$$

θ_d : is the phase deviation.

$\theta_d = k_p V_m$

$m_p = \theta_d$

* modulation index of PM $= \theta_d = k_p V_m$

$$\therefore S(t)_{PM} = A\cos\phi_i$$

$$\therefore S(t)_{PM} = A\cos\left[\omega_c t + \theta_d \cos\omega_m t\right]$$

also, the instantinuous frequency ω_i

$$\omega_i = \frac{d\phi_i}{dt} = \frac{d}{dt}\left[\omega_c t + k_p V_m \cos\omega_m t\right]$$

$$\omega_i = \omega_c - \underbrace{K_p V_m \omega_m}_{\text{max. deviation}} \sin\omega_m t$$

$$\therefore \boxed{\Delta\omega_{PM} = K_p V_m \omega_m}$$ depends on ω_m

* A relation between K_f & K_p can be developed for equal bandwidth of FM & PM,

since

$$\Delta\omega_{FM} = K_f V_m$$

$$\therefore \boxed{K_f = K_p \omega_m}$$

using Carson's rule :-

$$BW_{PM} \cong 2\Delta\omega \simeq 2K_p V_m \omega_m$$

Ex. A baseband signal $x(t) = 5\cos(2\pi \times 10^3 t)$ angle modulates a carrier signal $A_c \cos \omega_c t$.

① Determine the modulation index and bandwidth for (a) FM (b) PM.

② Find the change in the bandwidth and modulation index for both FM & PM if modulating frequency f_m is reduced to 5 KHz.

Assume $K_p = k_f = 15$ KHz/Volt.

Solution we have given $V_m = 5$ V & $f_m = 15$ KHz.

① For ⓐ FM system:-

Frequency deviation $\Delta f = k_f V_m = 15 \times 10^3 \times 5 = 75$ KHz

$$\therefore \quad m_f = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

$$BW_{fm} = 2(\Delta f + f_m) = 2(75 + 15) = 2 \times 90 = 180 \text{ KHz}$$

ⓑ For PM system

$$\Delta f = k_p V_m f_m = 15 \times 10^3 \times 5 \times 15 \times 10^3 = 1125 \text{ MHz}.$$

$$BW_{pm} = 2(\Delta f + f_m) \cong 2\Delta f = 2(1125 \text{ MHz}) = 2250 \text{ MHz}.$$

$$m_p = k_p V_m = 15 \times 10^3 \times 5 = 75{,}000$$

② Now the modulating frequency f_m reduced to 5 KHz.

ⓐ For FM $\quad m_f = \dfrac{\Delta f}{f_m} = \dfrac{75}{5} = 15$

$$BW_{fm} = 2(\Delta f + f_m) = 2(75 + 5) = 2 \times 80 = 160 \text{ KHz}.$$

ⓑ For PM $\quad \Delta f = k_p V_m f_m = 15 \times 10^3 \times 5 \times 5 \times 10^3 = 375 \text{ MHz}.$

$$BW = 2(\Delta f + f_m) \simeq 2\Delta f = 2 \times 375 = 750 \text{ MHz}.$$

$$m_p = k_p V_m = 15 \times 10^3 \times 5 = 75 \text{ KHz}.$$

Ex: Determine the following ① carrier and modulating frequencies ② modulation index and maximum deviation ③ the power dissipated in a $5\,\Omega$ resistor.

for an FM signal given as $v = 10 \sin[5\times10^{8}t + 4 \sin 1250t]$.

Solution

① $f_c = \dfrac{5\times10^{8}}{2\pi} = 79.57\ MHz$

② $f_m = \dfrac{1250}{2\pi} = 199\ Hz$

③ $m_f = 4$

④ $\Delta f = m_f f_m = 4 \times 199 = 796\ Hz$.

⑤ $P = \dfrac{[\text{RMS } \cancel{\text{of}} \text{ value of FM wave}]^2}{R} = \dfrac{\left[\frac{V_c}{\sqrt{2}}\right]^2}{R} = \dfrac{\left[\frac{10}{\sqrt{2}}\right]^2}{5} = 10\ W.$

EX. In an FM system, the modulating frequency $f_m = 1\ KHz$, the modulating voltage $V_m = 2$ volts and the deviation is $6\ KHz$. If the modulating voltage is raised to 4 volts, then, what will be the new deviation? If the modulating voltage is further increased to 8 volts and the modulating frequency is reduced to 500 Hz, what will be the deviation?

solution Given $f_m = 1\ KHz$, $V_m = 2\ V$, $\Delta f = 6\ KHz$

$\Delta f = K_f V_m \rightarrow K_f = \dfrac{\Delta f}{V_m} = \dfrac{6\ KHz}{2\ V} = 3\ KHz/v$

ⓐ when $V_m = 4\ V \rightarrow \Delta f = k_f V_m = 3\ \dfrac{kHz}{V} \times 4\ V = 12\ KHz$

ⓑ For $V_m = 8\ V$ ~~Δf~~ and $f_m = 500\ Hz$

$\Delta f = K_f V_m = 3\ \dfrac{kHz}{v} \times 8\ V = 24\ KHz$

* Find also the modulation index in each case.

ⓐ For $\Delta f = 6\ kHz$ & $f_m = 1\ KHz \rightarrow m_f = \dfrac{\Delta f}{f_m} = 6$

ⓑ $\Delta f = 12\ KHz$ & $f_m = 1\ KHz \rightarrow m_f = \dfrac{12}{1} = 12$

ⓒ $\Delta f = 24\ KHz$ & $f_m = 0.5\ kHz \rightarrow m_f = \dfrac{24}{0.5} = 48$

Ex. what will be the bandwidth required for an FM signal if the modulating frequency is 1 KHz & the maximum deviation is 10 KHz? what is the bandwidth required for the corresponding DSB (AM) transmission?

Solution $BW = 2[\Delta f + f_m] = 2[10+1] = 22\ KHz$.

The corresponding AM bandwidth is

$$BW_{AM} = 2f_m = 2\ KHz.$$

$\boxed{\text{* Note :- Carson's rule is valid as long as } m_f > 5 \text{ important.}}$

Ex. A 20 MHz carrier is modulated by a 400 Hz modulating signal. The carrier voltage is 5 V and the maximum deviation is 10 KHz. Write down the mathematical expression for the FM and PM waves. If the modulating frequency is increased to 2 KHz keeping every thing else constant, write down the expression for the FM & PM waves.

Solution

$$\omega_c = 2\pi \times 20 \times 10^6 = 1.25 \times 10^8 \text{ rad/sec.}$$

$$\omega_m = 2\pi \times 400 = 2513 \text{ rad/sec.}$$

$$m = m_f = m_p = \frac{\Delta f}{f_m} = \frac{10{,}000}{400} = 25$$

$\therefore$ FM-wave: $s(t)_{FM} = 5 \sin[1.25 \times 10^8 t + 25 \sin 2513t]$.

PM-wave: $s(t)_{PM} = 5 \sin[1.25 \times 10^8 t + 25 \sin 2513t]$

now, the $f_m = 2\ KHz \longrightarrow \omega_m = 2\pi \times 2000 = 12566.3$ rad/sec.

$$m_f = \frac{10{,}000}{2000} = 5$$

m_p is remains constant as it does not depend on the modulating frequency.

$\therefore m_p = 25$

Hence $\therefore$

$$s(t)_{FM} = 5[1.25 \times 10^8 t + 5 \sin 12566t]$$

$$s(t)_{PM} = 5[1.25 \times 10^8 t + 25 \sin 12566t].$$

EX. A carrier wave of amplitude of 10 V and frequency 100 MHz is frequency modulated by a sinusoidal voltage. The modulating voltage has an amplitude of 5 V and frequency $f_m = 20$ kHz. The frequency deviation constant is 2 kHz/volt. Draw the frequency spectrum of F.M wave. You have $J_0(0.5) \simeq 0.94$, $J_1(0.5) \simeq 0.24$, $J_2(0.5) \cong 0.03$

Solution modulating voltage $V_m = 5$ Volt

deviation constant $k_f = 2$ KHz/Volt

$\therefore$ frequency deviation $\Delta f = k_f \cdot V_m = 5 \times 2 \times 10^3 = 10^4$ Hz.

$$m_f = \frac{\Delta f}{f_m} = \frac{10^4}{20 \times 10^3} = 0.5.$$

Since we have $J_0 = 0.94$, $J_1 = 0.24$, $J_2 = 0.03$

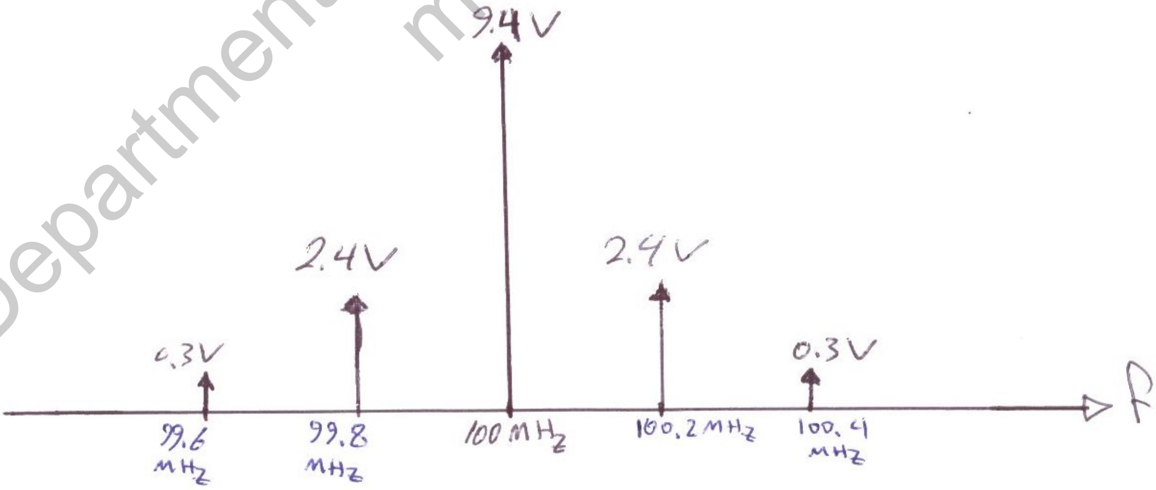
$\therefore$ carrier amplitude $= V_c J_0 = 10 \times 0.94 = 9.4$ volts.

Amplitude of the first pair of sidebands $= V_c J_1 = 10 \times 0.24 = 2.4$ volts.

Amplitude of the second pair of sidebands $= V_c J_2 = 10 \times 0.03 = 0.3$ volts.

The first pair of sidebands $f_c \pm f_m = 100$ MHz ± 20 KHz $= 100.2$ MHz & 99.8 MHz.

The second pair of sidebands $f_c \pm 2f_m = 100$ M ± 40 kHz $= 100.4$ MHz & 99.6 MHz.



EX. when the modulating frequency in an FM system is 400 Hz and the modulating voltage is 2.4 V, the modulation index is 60. calculate :-

① the maximum deviation ② what will be the modulation index when modulating frequency is reduced to 250 Hz and the modulating voltage is simultaneously raised to 3.2 V.

solution given $f_m = 400\,Hz$, $V_m = 2.4\,V$, $m_f = 60$.

ⓐ $\Delta f_{max} = m_f f_m = 24\,KHz$.

ⓑ $f_m = 250\,Hz$ & $V_m = 3.2\,V$

$\Delta f = K_f V_m \Rightarrow 24\,KHz = K_f \times 2.4 \Rightarrow K_f = 10\,KHz/volt$

$\therefore \Delta f = K_f V_m = 10 \times 3.2 = 32\,KHz$.

$$m_f = \frac{32 \times 10^3\,Hz}{250\,Hz} = 128.$$

EX. In An FM system, the audio frequency is 1 KHz and the audio voltage is 2 V. The deviation is 4KHz. If the Audio voltage increased to 8 V and the audio frequency droped to 500Hz, find the modulation index in each case and the corresponding bandwidth using Carson's rule.

Solution $f_{m1} = 1\,KHz$, $V_{m1} = 2\,V$, $\Delta f_1 = 4\,KHz$.

$f_{m2} = 0.5\,KHz$, $V_{m2} = 8\,V$.

$\Delta f_1 = k_{f_1} V_{m1} \rightarrow k_{f_1} = \frac{\Delta f_1}{V_{m_1}} = \frac{4 \times 10^3}{2} = 2 \times 10^3\,Hz/v$

$\Delta f_2 = k_f V_{m2} = 2 \times 10^3 \times 8 = 16\,KHz$.

$$m_{f_1} = \frac{\Delta f_1}{f_{m_1}} = \frac{4}{1} = 4$$

$$m_{f_2} = \frac{\Delta f_2}{f_{m2}} = \frac{16}{0.5} = 32.$$

$BW_1 = 2[\Delta f_1 + f_{m_1}] = 2\{4+1\} = 10\,KHz$.

$BW_2 = 2\{\Delta f_2 + f_{m_2}\} = 2\{16+0.5\} = 32\,KHz$.